On the role of electron-phonon interaction in the resistance anisotropy of two-dimensional electrons in GaAs heterostructures

2001 J. Phys.: Condens. Matter 13 11633

(http://iopscience.iop.org/0953-8984/13/50/322)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **13** (2001) 11633–11639 PII: S0953-8984(01)26240-7

On the role of electron–phonon interaction in the resistance anisotropy of two-dimensional electrons in GaAs heterostructures

D V Fil

Institute for Single Crystals, National Academy of Sciences of Ukraine, Lenin Avenue 60, Kharkov 61001, Ukraine

E-mail: fil@isc.kharkov.com

Received 26 June 2001, in final form 10 September 2001
Published 30 November 2001
Online at stacks.iop.org/JPhysCM/13/11633

Abstract

The contribution of the electron–phonon interaction to the energy of a unidirectional charge-ordered state (stripe phase) of two-dimensional electrons in GaAs heterostructures is analysed. The dependence of the energy on the direction of the electron-density modulation is calculated. It is shown that in electron layers situated close to the (001) surface the interference between the piezoelectric and the deformation potential interaction causes a preferential orientation of the stripes along the [110] axis.

1. Introduction

The observation of a resistance anisotropy of two-dimensional (2D) electrons at high Landau level occupancy [1–4] is considered as evidence for the formation of unidirectional charge-ordered states (stripe phases) in such systems. These states were predicted in [5–7]. The Hartree–Fock calculations [5–7] show that electrons at the topmost half-filled Landau level are separated into stripes of complete and zero occupancy with a period of several times the cyclotron radius. If the stripes are preferentially oriented along a certain direction, such states are expected to demonstrate anisotropy in the longitudinal resistance: low resistance along the stripes and high resistance in the perpendicular direction [8, 9].

The effect is observed in 2D electron layers in GaAs/AlGaAs heterostructures grown on (001)-oriented GaAs substrates. For such an experimental set-up in a magnetic field perpendicular to the electron layer, the low- and high-resistance directions are always pinned along certain crystallographic axes of the host matrix, namely, [110] and $[1\bar{1}0]$, respectively.

The mechanism that determines the preferential orientation of the stripes in GaAs heterostructures is not completely understood yet. It was suggested by Takhtamirov and Volkov [10] that an anisotropy of the effective mass of the 2D electrons may be responsible for the orientational pinning of the stripes. In the model developed in [10] the effective-mass

anisotropy is caused by an asymmetry of the quantum well potential confining the electrons to the (001) plane. This idea has been checked experimentally by Cooper *et al* [11]. In experiments [11] the influence of the symmetry of the quantum well potential on the resistance anisotropy has been studied. It has been found that the resistance does not depend on the precise form of the confining potential—in particular, the same form of the anisotropy is observed for symmetrically confining 2D electrons. The authors of [11] conclude that the asymmetry of the confining potential cannot be considered as an important factor.

Another possible mechanism for the stripe orientation has been studied by the present author [12]. It was shown that the piezoelectric interaction lowers the energy of the charge-density wave aligned along either the [110] or [1$\bar{1}$0] axes. Nevertheless, the piezoelectric mechanism alone does not explain why only one of two preferential orientations is realized. In this paper we address this problem. We extend the model proposed in [12] and take into account, together with the piezoelectric interaction, the deformation potential interaction. We find that, for the electron layers situated near the (001) surface of the sample, an interference between two channels of the electron–phonon coupling plays an important role in the stripe orientation. The mechanism considered explains the resistance anisotropy observed in the experiments.

2. The model

Let us consider a semi-infinite piezoelectric crystal that occupies a volume $z < 0$ and contains an electron layer at the distance d from the surface. The static energy of the system can be presented in the form

$$U = \int_{z<0} \mathrm{d}^3 r \left(\frac{\boldsymbol{E} \cdot \boldsymbol{D}}{8\pi} + \frac{u_{ik}\sigma_{ik}}{2} \right) + \int_{z>0} \mathrm{d}^3 r \, \frac{E^2}{8\pi} + U_{\mathrm{def}} \tag{1}$$

where

$$D_i = \varepsilon E_i - 4\pi \beta_{i,kl} u_{kl} \tag{2}$$

is the electric displacement field,

$$\sigma_{ik} = \lambda_{iklm} u_{lm} + \beta_{l,ik} E_l \tag{3}$$

is the stress tensor, $\boldsymbol{E}$ is the electric field, u_{ik} is the strain tensor, λ_{iklm} is the elastic moduli tensor, $\beta_{i,kl}$ is the piezoelectric moduli tensor, ε is the dielectric constant. To be more specific, we restrict our consideration to the case of a cubic lattice. The last term in (1) is the deformation potential interaction. It is chosen in the form

$$U_{\mathrm{def}} = \int \mathrm{d}^3 r \; \Lambda \rho (u_{xx} + u_{yy}) \delta(z + d) \tag{4}$$

where Λ is the deformation potential constant, ρ is the 2D electron density. Since we consider the model of an electron layer of zero thickness, the interaction with u_{zz} deformations is not included in (4).

The electric and elastic fields in equation (1) satisfy the following equations:

$$\nabla \boldsymbol{D} = 0 \tag{5}$$

$$\frac{\partial \sigma_{ik}}{\partial x_k} = 0 \tag{6}$$

(at $z < 0$) and

$$\nabla \boldsymbol{E} = 0 \tag{7}$$

(at $z > 0$).

At the free surface ($z = 0$) the boundary conditions are

$$D_z\big|_{z=-0} = E_z\big|_{z=+0} \tag{8}$$

$$E_{x(y)}\big|_{z=-0} = E_{x(y)}\big|_{z=+0} \tag{9}$$

$$\sigma_{iz}\big|_{z=-0} = 0. \tag{10}$$

At $z = -d$ the normal component of the electric displacement field is discontinuous:

$$D_z\big|_{z=-d+0} - D_z\big|_{z=-d-0} = 4\pi e\rho. \tag{11}$$

The deformation potential interaction induces a tangential force applied to the medium in the $z = -d$ plane. Rewriting the energy (4) in the form

$$U_{\text{def}} = -\int \mathrm{d}^2 r \sum_{i=x,y} u_i \Lambda\, \partial_i \rho \tag{12}$$

where $\boldsymbol{u}$ is the displacement vector, we obtain the following expression for the force, applied to unit area:

$$\boldsymbol{F} = \Lambda \left(\frac{\partial \rho}{\partial x}, \frac{\partial \rho}{\partial y}, 0 \right). \tag{13}$$

In equilibrium, this force is compensated by the stresses, and at $z = -d$ the stress tensor satisfies the boundary condition

$$\sigma_{iz}\big|_{z=-d+0} - \sigma_{iz}\big|_{z=-d-0} = -F_i. \tag{14}$$

Using equations (5)–(7) and the boundary conditions, one can reduce the energy (1) to the form

$$U = \tfrac{1}{2} \int \mathrm{d}^2 r\ (e\rho\varphi - u_i F_i) \tag{15}$$

where the electrostatic potential φ and the displacement field $\boldsymbol{u}$ are taken at $z = -d$. Their values are found from the solution of equations (5)–(7) under the boundary conditions specified.

3. The stripe-state energy anisotropy

Let us calculate the energy (15) for the stripe phase. We consider the system with the electron density modulated in a certain direction $\boldsymbol{r}_s$. The electron density can be presented as a Fourier series:

$$\rho(\boldsymbol{r}_{pl}) = \sum_{G_n} \mathrm{e}^{\mathrm{i}\boldsymbol{G}_n \cdot \boldsymbol{r}_{pl}} \rho_{\boldsymbol{G}_n} \tag{16}$$

where $\boldsymbol{G}_n = n\boldsymbol{q}$ (n is integer), $\boldsymbol{q} \parallel \boldsymbol{r}_s$, $|q| = 2\pi/l$, and l is the period of the stripe structure. For simplicity, we analyse the case of a unimodal charge-density wave:

$$\rho(\boldsymbol{r}_{pl}) = \rho_0 \cos \boldsymbol{q} \cdot \boldsymbol{r}_{pl}. \tag{17}$$

We calculate the energy (15) as a series in powers of the electron–phonon interaction constants:

$$\frac{U}{S} = U_0 + U_2 + \cdots \tag{18}$$

where S is the area of the electron layer,

$$U_0 = \frac{\pi e^2 \rho_0^2}{2q\varepsilon} \left(1 + \frac{\varepsilon - 1}{\varepsilon + 1} \mathrm{e}^{-2qd} \right) \tag{19}$$

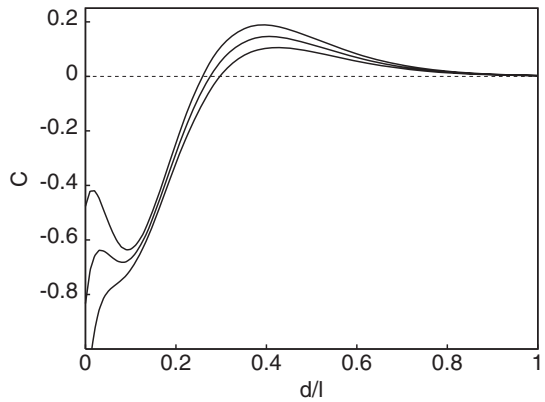

Figure 1. The parameter of the stripe energy anisotropy (equation (22)) in units of E_i for the isotropic crystal; $\eta = 2.7, 2.5, 2.3$ (from top to bottom).

is the Coulomb energy in the absence of electron–phonon interaction, and the term U_2 is quadratic in the interaction constants. For the concrete physical systems considered below, the contribution of the higher-order terms in (18) is very small and it can be neglected. The term U_2 determines the dependence of the energy on the direction of $\boldsymbol{q}$.

Let us first consider an isotropic crystal, for which the sound velocities do not depend on the direction of the sound propagation. For a cubic lattice this condition is realized if the elastic constants satisfy the relation $c_{11} - c_{12} - 2c_{44} = 0$. For such a special case, an analytical expression for U_2 can be presented in a simple form. For the (001) electron layer the calculation of U_2 gives the following result:

$$U_2 = A + B\cos 4\psi + C\sin 2\psi \tag{20}$$

where ψ is the angle between $\boldsymbol{q}$ and the [100] axis. The angle dependence (20) is determined by the parameters $\eta = c_{11}/c_{44}$ and $\xi = qd$. Using the strong inequality $\epsilon \gg 1$, that applies for GaAs, we find for B and C the following approximate expressions:

$$B = E_p\left[1 - \frac{\eta}{3} - \mathrm{e}^{-2\xi}\left(\frac{(\eta-3)[2\eta(1-2\xi)+\xi^2(5\eta-3)]}{9(\eta-1)} + \xi^3\left[\frac{2}{3}(\eta+1) - \xi(\eta-1)\right]\right)\right] \tag{21}$$

$$C = E_i\mathrm{e}^{-2\xi}\left(\frac{\eta(\eta-3)(1-\xi)}{\eta-1} + \xi^2[3\xi(\eta-1) - 2\eta - 3]\right) \tag{22}$$

where $E_p = 9\pi^2 e^2\rho_0^2 e_{14}^2/64\varepsilon^2 c_{11}q$, $E_i = \pi|e|\rho_0^2 e_{14}\Lambda/8\varepsilon c_{11}$. We do not present here the expression for the parameter A, which does not influence the orientation.

In equation (20) the second term describes the anisotropy determined by the piezoelectric interaction, and the third term describes the anisotropy caused by the interference of the piezoelectric and the deformation potential interactions. One can see that at $\xi \to \infty$ the interference term tends to zero and the energy (20) has C_{4v} symmetry. If $\xi \sim 1$, the interference term is essential and the C_{4v} symmetry is reduced to C_{2v}. We find that at $\eta < 2.7$ the parameter B is positive for all ξ and the global minimum is reached at $\psi_m = \pi/4$ or $\psi_m = 3\pi/4$ depending on the sign of the parameter C. At $C < 0$, $\psi_m = \pi/4$ and the stripes are preferentially oriented along the $[1\bar{1}0]$ axis, while at $C > 0$, $\psi_m = 3\pi/4$ and the [110]-oriented stripe phase has the lowest energy. The dependence of C on $d/l = \xi/2\pi$ is shown in figure 1. One can see that at $d/l > 0.3$ the [110] orientation of the stripes is realized.

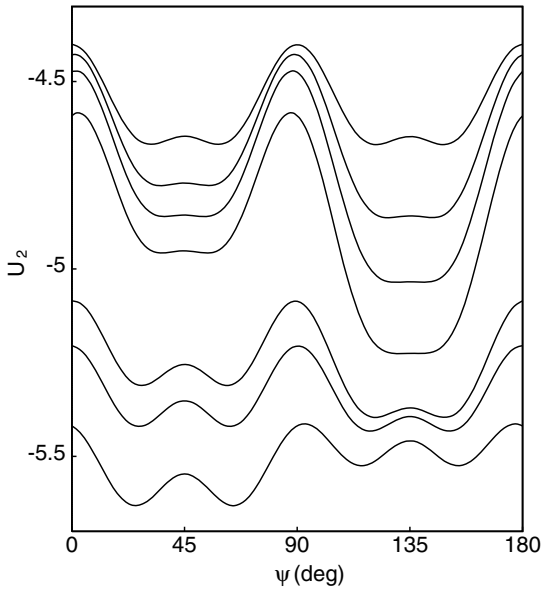

Figure 2. The phonon contribution to the energy of the stripe structure (in units of E_p) for GaAs versus the direction of the electron-density modulation at $d/l = 1.0, 0.6, 0.5, 0.4, 0.25, 0.23, 0.2$ (from top to bottom).

The results obtained are sensitive to the parameters of the system. Therefore, to determine the stripe orientation in GaAs heterostructures it is necessary to take into account the anisotropy of the elastic moduli. For this case we solve equations (5)–(7), with the boundary conditions specified, numerically. The following parameters are used for the calculations: $c_{11} = 12.3 \times 10^{10}\,\mathrm{N\,m^{-2}}$, $c_{12} = 5.4 \times 10^{10}\,\mathrm{N\,m^{-2}}$, $c_{44} = 6.0 \times 10^{10}\,\mathrm{N\,m^{-2}}$, $e_{14} = 0.15\,\mathrm{C\,m^{-2}}$ $\Lambda = 7.4$ eV, $\varepsilon = 12.5$. The dependence of U_2 on the direction of $\boldsymbol{q}$ at $l = 2 \times 10^3$ Å and several d/l is shown in figure 2. Unlike in the case for the isotropic crystal, the minima of the energy are reached at q deviating from the [110] or [1$\bar{1}$0] axes by the angles $\Delta\psi \approx \pm\pi/12$. If d/l is in the interval [0.23, 1], the configurations with a small deviation of q from the [1$\bar{1}$0] axis have the lowest energy. In this case one can expect a domain structure to form, and, on average, the system should show the minimum resistance in the [110] direction and the maximum resistance in the perpendicular direction. The calculations predict the largest resistance anisotropy at $d/l = 0.4$. At larger and at smaller d/l the anisotropy becomes weaker. At $d/l > 1$ and $d/l \approx 0.23$ it disappears completely. At $d/l < 0.23$ the resistance anisotropy is reset, but the high- and the low-resistance directions alternate.

4. Discussion

We have shown that the classical energy of a charge-density wave in a 2D electron system embedded in a piezoelectric matrix depends on the direction of the wave vector. The effect is caused by the electron–phonon interaction. The minimum energy is reached in two or four different directions of the density modulation. Therefore, the stripes may form a polydomain structure. In the bulk samples, polydomain structures may not show any resistance anisotropy, while in 2D layers situated near the surface of the sample, the resistance anisotropy should occur.

We calculate the static energy of the stripe structure. Our results can be also understood as an effect of a virtual exchange of acoustic phonons between electrons. The electron–electron interaction in bulk isotropic piezoelectrics caused by the virtual exchange of acoustic phonons has been studied by Rashba and Sherman [13]. Our approach [12] (see also [14], where the orientation of bi-layer Wigner crystals has been studied) reproduces the results of [13]. Within such an interpretation the effect described in the present paper is a consequence of the virtual exchange of surface phonon modes. For the surface acoustic waves on the (001) surface, the piezoelectric and deformation potential interactions give in-phase contributions to the matrix elements of the electron–phonon interaction [15], and the interference between two channels of electron–phonon interaction takes place.

The unimodal approximation predicts that at $d/l > 0.23$ the monodomain stripe structure has the lowest energy for stripes deviating from the [110] axis by the angle $\Delta\psi \approx +\pi/12$ or $-\pi/12$. In this case, the polydomain structure should show an anisotropic resistance with a minimum in the [110] direction. The largest resistance anisotropy is reached at $d/l \approx 0.4$. At $d/l > 1$ the anisotropy becomes exponentially weak. At $d/l < 0.23$ the anisotropy changes its sign. Note that the last conclusion is specific to the unimodal approximation (17). The higher harmonics in (16) will shift the transition point to a smaller value of d/l.

We consider that our model describes the orientation of stripe structures formed at high Landau levels. We analyse the anisotropy of the direct interaction between electrons. One can expect the anisotropy of the exchange interaction to be small, and not to influence the orientation significantly.

In the Hartree–Fock approximation the period of the electron-density modulation l is approximately equal to $6\ell_H$, where ℓ_H is the magnetic length. In experiments [11], the resistance anisotropy was observed at the magnetic field $H \approx 2$ T and for $d \approx 2 \times 10^3$ Å. Therefore, for the Hartree–Fock l we find the ratio $d/l \approx 2$. At such d/l the surface effects are not important and only an exponentially small violation of the C_{4v} symmetry may take place. Since in experiments this violation is quite large, we suppose that the period l is larger than the one given by the Hartree–Fock theory.

It is interesting to evaluate the absolute value of the native anisotropy caused by the phonon mechanism. Using the parameters given before for $d/l = 0.4$, the filling factor $\nu = 9/2$, the electron density $n = 2 \times 10^{11}$ cm^{-2}, and $\rho_0 \approx \bar{\rho}$ (where $\bar{\rho}$ is the average density at the valence Landau level), we find the anisotropy energy $E_a \approx 0.7$ mK per electron (we determine E_a as the energy for $\boldsymbol{q} \parallel [110]$ minus the energy for $\boldsymbol{q} \parallel [1\bar{1}0]$).

In our consideration we neglect the screening of the electron–phonon coupling caused by the polarization of the remote Landau levels. To evaluate the effect of screening in the unimodal approximation, one can use the effective dielectric constant $\varepsilon_{\rm eff}(q) = \varepsilon(1 + K_q)$ and the screened deformation potential $\Lambda_{\rm scr}(q) = \Lambda/(1 + K_q)$, given by the random-phase approximation. Here

$$K_q = \frac{e^2 q}{\varepsilon\omega_c} \exp\left(-\frac{q^2\ell_H^2}{2}\right) \sum_{n,m,\sigma}{}' \frac{n!}{m!} \frac{f_{n\sigma}(1 - f_{m\sigma})}{m - n} \left(\frac{q^2\ell_H^2}{2}\right)^{m-n-1} \left[L_n^{m-n}\left(\frac{q^2\ell_H^2}{2}\right)\right]^2. \quad (23)$$

In equation (23), $f_{n\sigma} = n_F(\varepsilon_{n\sigma})$ is the Fermi factor, ω_c is the cyclotron frequency, L_n^m is the generalized Laguerre polynomial, and the prime on the sum excludes the valence Landau level (compare with [16]). With such a substitution, the form of the dependence $U_2(\psi)$ (figure 2) remains almost unchanged, but the absolute value of the anisotropy is reduced to $E_a^{\rm src} \approx E_a/(1 + K_q)^2$. Evaluation of the formula (23) for the parameters given above yields $E_a^{\rm src} \approx 0.4$ mK.

The phonon contribution to the native anisotropy is comparable with the one given by the effective-mass anisotropy mechanism [10] (it gives a value of order of 1 mK). The experimental

data for the anisotropy energy have been obtained in [11] from the measurements of the resistivity in a tilted magnetic field. The data presented in [11] are based on theoretical calculations by Jungwirth *et al* [16]. According to [11] the anisotropy energy for the sample with a conventional heterostructure is higher (2.4 mK) than for the sample with a symmetric quantum well (0.5 mK). Thus, we conclude that the two mechanisms of the anisotropy work in parallel, and the survival of the anisotropy in samples with symmetric quantum wells can be accounted for by the phonon mechanism.

The mechanism of the resistance anisotropy considered is this paper is fundamentally dependent on the distance between the surface of the sample and the electron layer. Therefore, it is desirable to investigate this dependence experimentally. Such a study may answer the question of whether or not the surface effects play an important role in the stripe orientation.

It is also of interest to investigate experimentally the influence of the electron layer orientation on the resistance anisotropy. To illustrate this point, we outline the results obtained for the (111) layer in the isotropic crystal. For such a system the energy (20) is modified to

$$u_2 = A' + B' \cos 6\psi \tag{24}$$

where ψ is measured from the $[0\bar{1}1]$ axis. The coefficient B' is given by the following expression:

$$B' = E_p \frac{10}{27}\left(1 - \eta - \frac{\mathrm{e}^{-2\xi}}{5(\eta-1)}[4(\eta^2-\eta-1)(1+2\xi) + \xi^2(\eta+1)(5\eta-7) + \tfrac{2}{3}\xi^3(\eta+1)(\eta-5) - \xi^4(\eta-1)^2]\right). \tag{25}$$

It is important to note that the interference term in (24) does not depend on ψ (it is included in A'). We find that the parameter B' is negative for all ξ, and $\eta > 2$. Therefore, the minimum of the energy is reached at $\psi_m = n\pi/3$ (n is an integer). This means that monodomain stripe structures can show low resistance along any of the $[\bar{2}11]$, $[1\bar{2}1]$, and $[11\bar{2}]$ directions, while the polydomain structures may not show any resistance anisotropy at all. Since this conclusion is not sensitive to the parameters of the systems, the lattice anisotropy is not very fundamental in this case, and the same behaviour is expected for the GaAs system.

References

[1] Lilly M P, Cooper K B, Eisenstein J P, Pfeiffer L N and West K W 1999 *Phys. Rev. Lett.* **82** 394
[2] Du R R, Tsui D C, Stormer H L, Pfeiffer L N, Baldwin K W and West K W 1999 *Solid State Commun.* **109** 389
[3] Pan W, Du R R, Stormer H L, Tsui D C, Pfeiffer L N, Baldwin K W and West K W 1999 *Phys. Rev. Lett.* **83** 820
[4] Lilly M P, Cooper K B, Eisenstein J P, Pfeiffer L N and West K W 1999 *Phys. Rev. Lett.* **83** 824
[5] Koulakov A A, Fogler M M and Shklovskii B I 1996 *Phys. Rev. Lett.* **76** 499
[6] Fogler M M, Koulakov A A and Shklovskii B I 1996 *Phys. Rev.* B **54** 1853
[7] Moessner R and Chalker J T 1996 *Phys. Rev.* B **54** 5006
[8] Oppen F, Halperin B I and Stern A 2000 *Phys. Rev. Lett.* **84** 2947
[9] MacDonald A H and Fisher M P A 2000 *Phys. Rev.* B **61** 5724
[10] Takhtamirov E E and Volkov V A 2000 *Pis. Zh. Eksp. Teor. Fiz.* **71** 712 (Engl. transl. 2000 *JETP Lett.* **71** 422)
[11] Cooper K B, Lilly M P, Eisenstein J P, Jungwirth T, Pfeiffer L N and West K W 2001 *Solid State Commun.* **119** 89
[12] Fil D V 2000 *Fiz. Nizk. Temp.* **26** 792 (Engl. transl. 2000 *Low Temp. Phys.* **26** 581)
(Fil D V 2000 *Preprint* cond-mat/0004107)
[13] Rashba E I and Sherman E Ya 1987 *Fiz. Tekh. Poluprov.* **21** 1957 (Engl. transl. 1987 *Sov. Phys.–Semicond.* **21** 1185)
[14] Fil D V 2001 *Fiz. Nizk. Temp.* **27** 523 (Engl. transl. 2001 *Low Temp. Phys.* **27** 384)
(Fil D V 2001 *Preprint* cond-mat/0104438)
[15] Knabchen A, Levinson Y B and Entin-Wohlman O 1996 *Phys. Rev.* B **54** 10 696
[16] Jungwirth T, MacDonald A H, Smrcka L and Girvin S M 1999 *Phys. Rev.* B **60** 15 574